Propagation of random errors

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Linear combination

Propagation of the error in the linear combination of

independent measurements

2. ’l 1 .’| Linear combinations

In this case the final value, y, is calculated from a linear combination of measured

quantities u, b, c, etc., by:

y=l<+I<,,u+k,,b+k,c+--~ (2.10)

where k, ka, kb, ku etc., are constants. Variance (defined as the square of the standard

deviation) has the important property that the variance of a sum or difference of

independent quantities is equal to the sum of their variances. It can be shown that if

0;,, 0,,, og, etc., are the standard deviations of u, b, c, etc., then the standard deviation

of y, 0,, is given by:

cr, = ¤l(l<i.¤">’ + (kh¤.>“ + <k,¤J\* + - · — (2.11)

Illustratron rn R

Illustratrcn usrng random samples generated by R:

kqenerarwrq data

#1r11t1al xeadnrtq

><=:nc1Y41OG,3.3l,C.El2T

vrixnal xeadnnq

y—rn¤¤‘v\*4lOO,l5.67,U.U2)

Qivolume used

rnesnrx)

me¤u1(Y)

mean (2)

$dl><)

S¤ly>

sdiz)

sq[L{vaz1><)+va:(y7Y

l\/lultiplicative expressions

The following is true only approximately. Still measurements

are assumed to be independent.

2.1 1.2 Multiplicative expressions

Ify is calculated from an expression ofthe type;

y ; kub/cz! (2.12)

(where rz, b, c and rl are independent measured quantities and k is a constant) then

there is a relationship between the squares of the relative standard deviations:

Lllle) t {el) (2,13,

}(y /1 ·> /1 as e /“

Exam ple

Example 2/l’|.2 . - .» \_ -·

The quantum yield of fluorescence, ¢>, is calculated from the expression:

Q) = Qkclloe)

where the quantities involved are defined below, with an estimate of th

relative standard deviations in brackets:

In = incident light intensity (0.5%)

I): fluorescence intensity (2%)

s = molar absorptivity (1%)

c = concentration (0.2%)

I : path-length (0.2%)

k is an instrument constant.

From equation (2.13), the relative standard deviation of o is given by:

RSD : V22 + 0.22 + 0.22 + 0.52\*+-? = 2.3%

Illustration in R

Illustration using random samples generated by R:

llganazactrg data

#1nc1der¤L hqlnt xntenszty

IO:¤1crrNlOOilLZ,U.UU5—QUl

T\*:rn0rrMlOOi l5yU4U}·`$l

ltmular absoxpznvnty

e=rncrmllO(l,5,C.Ul¤El

ltccncentxatnori

c;rn0r\*nllOUi2iU.CU2—2l

dpaliwflenqth

L:rm>L'PllOOill,U.(I02~ill

Hjovnputixug quaniurw y:e—0 of Uqcrescertce

pm=1f/l¤:.1.1G,el

s¤(><) fynearlplul

2.’|E.'| Lmear cumbinatiuns

2.'I 2.2 Mulmplncatnva zxpressinns

[Ay/H : we/ui + (Ah/D) + um] ~ [Ad/J) (2.1Sv